The First Note book

First prep

The Rational Numbers

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Set of Rational Numbers

Remember:-

① Counting Numbers = $\{1, 2, 3, 4, \dots\}$

② Set of Integers $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

③ Set of Natural Number = $\{0, 1, 2, 3, \dots\}$

④ $Z^+ = \{1, 2, 3, \dots\}$

⑤ $Z^- = \{-1, -2, -3, \dots\}$

⑥ $N \subset Z$

⑦ $Z = Z^+ \cup \{0\} \cup Z^-$

⑧ If $|x| = a$ then $x = \pm a$

Ex 1:- Complete

① $Z - N = \dots$

② $Z^+ \cup \{0\} = \dots$

③ $Z^+ \cap Z^- = \dots$

④ $N \cap Z =$

⑤ $|-3| = \dots$

⑥ $|0| = \dots$

⑦ If $|x| = 7$ then $x = \dots$ or $x = \dots$

⑧ If $|x| = 0$ then $x = \dots$

Ex 2: find the value of x when

① $x + 2 = |-3|$

② $|x| + 2 = 5$

③ $|x| - 5 = 2 + |-2|$

④ $2x + 5 = 7$

③

Definition of the rational number:-

It's a number that can be expressed in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$

i.e.

$$Q = \left\{ x : x = \frac{a}{b} , a \in Z , b \in Z , b \neq 0 \right\}$$

EX:- put $\in$ or $\notin$

		the answer		Because
① $\frac{2}{3}$ ...	Q	$\in$	$\frac{2}{3}$	in the form $\frac{a}{b}, b \neq 0$
② 0.2 ...	Q	$\in$	$0.2 = \frac{2}{10}$	in the form $\frac{a}{b}, b \neq 0$
③ 25% ...	Q	$\in$	$25\% = \frac{25}{100}$	in the form $\frac{a}{b}$
④ $1\frac{1}{4}$ ...	Q	$\in$	$1\frac{1}{4} = \frac{5}{4}$	in the form $\frac{a}{b}$
⑤ -2 ...	Q	$\in$	$-2 = \frac{-2}{1}$	
⑥ zero ...	Q	$\in$	zero $= \frac{0}{1}$	
⑦ 3 ...	Q	$\in$	$3 = \frac{3}{1}$	
⑧ $\frac{1}{0}$ ...	Q	$\notin$		be cause the denominator $= 0$
⑨ $\frac{2}{5-5}$ ...	Q	$\notin$		be cause the denominator $= 0$

Notes

① $\frac{a}{b} \in Q$ if $b \neq 0$ (denominator $=$ zero)

② $\frac{a}{b} = 0$ if $a =$ zero , $b \neq$ zero

③ $\frac{a}{b}$ is an integer if the numerator is divisible by the denominator.

④ $\dfrac{a}{b} > 0$ (positive) if $ab > 0$

(if a, b have the same sign)

⑤ $\dfrac{a}{b} < 0$ (negative) if $ab < 0$

Ex:- If x is an integer, write the required condition to make each of the following a rational number:

① $\dfrac{2}{x-5}$ ⟹ the Condition $x-5 \neq 0$ i.e $x \neq 5$

② $\dfrac{5}{x+3}$ ⟹ the Condition $x+3 \neq 0$ i.e $x \neq -3$

③ $\dfrac{7}{2x-4}$ ⟹ the Condition $2x-4 \neq 0 \gg 2x \neq 4 \quad \div 2$

i.e. $x \neq 2$

④ $\dfrac{5}{2x}$ ⟹ the Condition $2x \neq 0 \div 2$ i.e $x \neq 0$

⑤ $\dfrac{3}{|x|-4}$ ⟹ the Condition $|x|-4 \neq 0 \Rightarrow |x| \neq 4$

i.e. $x \neq \pm 4$

⑥ $\dfrac{2}{3x+6}$ ⟹ the Condition $3x+6 \neq 0$

therefore ⟹ $3x \neq -6 \quad \div 3$

i.e. $x \neq -2$

Ex:- If x is an integer, write the required condition to make each of the following $=$ zero

$\boxed{1}$ $\dfrac{x-3}{4}$ since $\dfrac{x-3}{4} = 0$ therefore $x - 3 = 0$

i.e $x = 3$

$\boxed{2}$ $\dfrac{2-x}{3}$

Since $\dfrac{2-x}{3} = 0$ there fore $2 - x = 0$ i.e $x = 2$

$\boxed{3}$ $\dfrac{x+3}{x+4}$

Since $\dfrac{x+3}{x+4} = 0$ therefore $x + 3 = 0$ i.e. $x = -3$

$\boxed{4}$ $\dfrac{3x-3}{x}$

Since $\dfrac{3x-3}{x} = 0$ therefore $3x - 3 = 0$

$\Rightarrow 3x = 3 \div 3$ i.e $x = 1$

$\boxed{5}$ $\dfrac{2x}{x+3}$

Since $\dfrac{2x}{x+3} = 0$ therefore $2x = 0 \div 2 \Rightarrow x = 0$

$\boxed{6}$ $\dfrac{3x+6}{x+1}$

Since $\dfrac{3x+6}{x+1} = 0$ there fore

$3x + 6 = 0$

$3x = -6 \div 3$

$\boxed{x = -2}$

Different forms of a rational number

1) writing the rational number in its simplest form:

Ex: put each of the following numbers in its simplest form:

1) $\frac{10}{15} = \frac{2}{3}$ $(\div 5)$ Dividing the two terms by 5

2) $\frac{45}{20} = \frac{9}{4}$ $(\div 5)$

3) $|-\frac{18}{12}| = \frac{18}{12} = \frac{3}{2}$ Dividing the two Terms by 6

4) $|20\%| = \frac{20}{100} = \frac{1}{5}$ Dividing the two terms by 20

5) $-\frac{24}{28} = -\frac{6}{7}$ Dividing the two terms by 4

* **the form of a terminating decimal**

To write the rational number in the form of a terminating decimal we mak its denominator equal to 10, 100, 1000, ...

Ex: write each of the following numbers in the form of a terminating decimal:

1) $\frac{2}{5}$ (To mak denominator 10 multiplying the two terms by 2

$\frac{2}{5} = \frac{4}{10} = 0.4$

② $\dfrac{-3}{8}$

To make denominator 1000 multiplying the two Terms by 125)

$$\dfrac{-3}{8} = \dfrac{-3 \times 125}{8 \times 125} = -\dfrac{375}{1000} = -0.375$$

③ $\dfrac{3}{4}$

To make denominator 100 multiplying the two Terms by 25

$$\dfrac{3}{4} = \dfrac{3 \times 25}{4 \times 25} = \dfrac{75}{100} = 0.75$$

④ $\dfrac{7}{25}$

To make denominator 100 multiplying the two terms by 4

$$\dfrac{7}{25} = \dfrac{7 \times 4}{25 \times 4} = \dfrac{28}{100} = 0.28$$

⑤ $-2\dfrac{2}{25} = -2\dfrac{2 \times 4}{25 \times 4} = -2\dfrac{8}{100} = -2.08$

⑥ $\dfrac{7}{5} = \dfrac{7 \times 2}{5 \times 2} = \dfrac{14}{10} = 1.4$

⑦ $|-2\tfrac{1}{5}| = 2\tfrac{1}{5} = 2\dfrac{1 \times 2}{5 \times 2} = 2\dfrac{2}{10} = 2.2$

⑧ $-5\dfrac{3}{5} = -5\dfrac{3 \times 2}{5 \times 2} = -5\dfrac{6}{10} = -5.6$

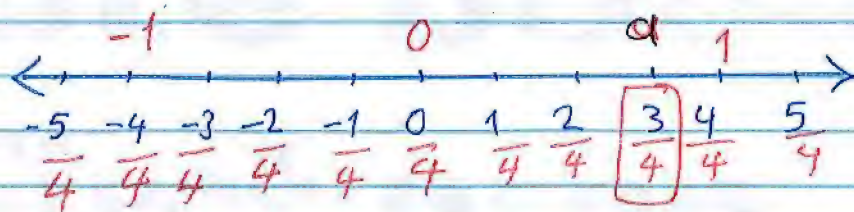* the form of percentage :

To write the rational number in the form of percentage we make its denominator = 100

Ex: Write each of the following numbers in the form of percentage

1) $\frac{3}{5} = \frac{3}{5} \times 100\% = 60\%$

2) $\frac{7}{5} = \frac{7}{5} \times 100\% = 140\%$

3) $\frac{1}{2} = \frac{1}{2} \times 100\% = 50\%$

4) $\frac{5}{4} = \frac{5}{4} \times 100\% = 125\%$

5) $|-\frac{6}{25}| = \frac{6}{25} \times 100\% = 24\%$

6) $3.2 = 3.2 \times 100\% = 320\%$

7) $0.15 = 0.15 \times 100\% = 15\%$

8) $\frac{9}{20} = \frac{9}{20} \times 100\% = 45\%$

The recurring decimal

Ex:- Write each of the following in the form $\frac{a}{b}$

1) $0.\dot{5} = \frac{5}{9}$

2) $-0.\dot{7} = -\frac{7}{9}$

3) $0.\dot{4}\dot{5} = \frac{45}{99} = \frac{5}{11}$

4) $-0.\dot{1}\dot{2} = -\frac{12}{99} = -\frac{4}{33}$

5) $0.\dot{4}0\dot{7} = \frac{407}{999}$

6) $9.\dot{3} = 9\frac{3}{9} = 9\frac{1}{3}$

Ex:- Write each of the following in the form $\frac{a}{b}$

1) $0.4 = \frac{4}{10} = \frac{2}{5}$

2) $0 = \frac{0}{1}$

3) $-0.01 = -\frac{1}{100}$

4) $0.75 = \frac{75}{100} = \frac{3}{4}$

5) $3\frac{3}{4} = \frac{15}{4}$

6) $8\frac{2}{3} = \frac{26}{3}$

7) $0.001 = \frac{1}{1000}$

8) $-7 = \frac{-7}{1}$

9) $1.25 = \frac{125}{100} = \frac{25}{20} = \frac{5}{4}$

10) $30\% = \frac{30}{100} = \frac{3}{10}$

11) $45\% = \frac{45}{100} = \frac{9}{20}$

12) $0.\dot{5} = \frac{5}{9}$

13) $0.\dot{1}\dot{5} = \frac{15}{99} = \frac{5}{33}$

14) $-1.\dot{1}\dot{8} = -1\frac{18}{99} = -1\frac{2}{11} = \frac{-13}{11}$

15) $0.0\dot{4}\dot{5} = \frac{45}{990} = \frac{5}{110} = \frac{1}{22}$

16) $0.\dot{2}8\dot{5} = \frac{285}{999} = \frac{95}{333}$

Try by yourself

1) write the required condition to make each of the following a rational number:

(1) $\dfrac{5}{a-3}$ (2) $\dfrac{3}{4-x}$ (3) $\dfrac{7}{8b}$ (4) $\dfrac{2}{|x|-1}$

(5) $\dfrac{5}{x}$ (6) $\dfrac{6}{x-9}$ (7) $\dfrac{2}{5x-10}$ (8) $\dfrac{4}{2x+8}$

(9) $\dfrac{1}{5-x}$ (10) $\dfrac{2}{3a+3}$ (11) $\dfrac{16}{4-|x|}$ (12) $\dfrac{8x}{3|x|-9}$

2) Write the required condition to make each of the following $= 0$

(1) $\dfrac{a-3}{a+1}$ (2) $\dfrac{b+2}{b-2}$ (3) $\dfrac{4-x}{x}$ (4) $\dfrac{5+y}{y+7}$

(5) $\dfrac{2y-4}{y-1}$ (6) $\dfrac{6-3x}{x+4}$ (7) $\dfrac{x}{x+5}$ (8) $\dfrac{2x}{x-3}$

3) put each of the following numbers in the form of a terminating decimal:

(1) $\dfrac{1}{2}$ (2) $\dfrac{3}{5}$ (3) $\dfrac{-7}{20}$ (4) $2\dfrac{3}{25}$ (5) $\dfrac{3}{2}$ (6) $\dfrac{7}{2}$

(7) $\dfrac{12}{5}$ (8) $\dfrac{-6}{25}$ (9) $\dfrac{-7}{50}$ (10) $\dfrac{5}{8}$ (11) $-2\dfrac{3}{5}$

4) write each of the following numbers in the form of percentage:

(1) $\dfrac{4}{5}$ (2) $\dfrac{5}{8}$ (3) $2\dfrac{1}{2}$ (4) $\dfrac{8}{25}$ (5) $\dfrac{1}{4}$ (6) $1-\dfrac{1}{5}$

(7) $|-0.4|$ (8) $\dfrac{6}{4}$ (9) $\left|\dfrac{-13}{2}\right|$ (10) $0.\dot{3}$

Comparing and ordering Rational Numbers.

Ex.1 Represent the rational number $\frac{3}{4}$ on the number line

Solution:-

$$\frac{-5}{4} \quad \frac{-4}{4} \quad \frac{-3}{4} \quad \frac{-2}{4} \quad \frac{-1}{4} \quad \frac{0}{4} \quad \frac{1}{4} \quad \frac{2}{4} \quad \boxed{\frac{3}{4}} \quad \frac{4}{4} \quad \frac{5}{4}$$

Ex.2: Represent the rational numbers $2, 0, \frac{3}{2}, \frac{-5}{2}, -3$

on the number line then arrange them descendingly

Solution:
We should convert their denominators to have common denominator at first

Since L.C.M of the denominators is 2
Then the numbers after converting their denominator are

$$\frac{4}{2} \;,\; \frac{0}{2} \;,\; \frac{3}{2} \;,\; \frac{-5}{2} \;,\; \frac{-6}{2}$$

$$\frac{-7}{2} \;\; \frac{-6}{2} \;\; \frac{-5}{2} \;\; \frac{-4}{2} \;\; \frac{-3}{2} \;\; \frac{-2}{2} \;\; \frac{-1}{2} \;\; \frac{0}{2} \;\; \frac{1}{2} \;\; \frac{2}{2} \;\; \frac{3}{2} \;\; \frac{4}{2} \;\; \frac{5}{2} \;\; \frac{6}{2}$$
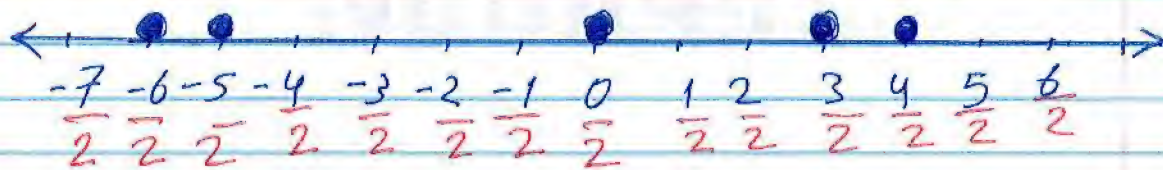
then ; $\frac{4}{2} > \frac{3}{2} > \frac{0}{2} > \frac{-5}{2} > \frac{-6}{2}$

i.e $2 > \frac{3}{2} > 0 > \frac{-5}{2} > -3$

Ex3 Compare between the two rational numbers

$$\frac{2}{3} \quad \text{and} \quad \frac{3}{4}$$

Solution

We should convert their denominators to have common denominator at first

$$\frac{2}{3} \times \frac{3}{4} \implies \frac{8}{12} \quad \frac{9}{12}$$

Since $\frac{8}{12} < \frac{9}{12}$ then $\frac{2}{3} < \frac{3}{4}$

Ex4: find four rational numbers lying between

the two numbers ① $\frac{1}{2}$ and $\frac{1}{3}$

Solution   ② $\frac{-3}{5}$ and $\frac{-2}{3}$

① $\frac{1}{2} \times \frac{1}{3} \implies \frac{3}{6}$ and $\frac{2}{6}$

$$\implies \frac{30}{60} \quad \text{and} \quad \frac{20}{60}$$

$$\implies \frac{20}{60} < \frac{21}{60} < \frac{22}{60} < \frac{23}{60} < \frac{24}{60} < \frac{30}{60}$$

② $\frac{-3}{5} \times \frac{-2}{3} \implies -\frac{9}{15} \quad -\frac{10}{15}$

$$\implies \frac{-90}{150} \quad -\frac{100}{150}$$

then the numbers are: $-\frac{91}{100}$, $-\frac{92}{100}$, $-\frac{93}{100}$, $-\frac{94}{100}$

Ex:5 Complete each of the following using the suitable sign (< , > or =):

① $\frac{7}{5}$ ... $\frac{4}{5}$ ② $-\frac{3}{4}$ ... $-\frac{2}{4}$

③ $\frac{1}{5}$ ... $\frac{1}{6}$ ④ $\frac{3}{6}$ ... $\frac{2}{3}$

⑤ $\frac{4}{10}$ ... $\frac{14}{35}$ ⑥ $|-\frac{10}{15}|$ ... $\frac{2}{3}$

Try by yourself

① Represent on the number line each of the following rational numbers

① $\frac{1}{3}$ ② $-\frac{7}{4}$ ③ $-\frac{1}{3}$ ④ $1\frac{1}{5}$ ⑤ $-3\frac{1}{2}$

② Compare between

① $\frac{2}{3}$ and $\frac{5}{7}$ ② $\frac{1}{5}$ and $\frac{1}{6}$ ③ $-\frac{8}{15}$ and $-\frac{2}{3}$

④ 0.6 and $\frac{5}{6}$

③ find two rational numbers lying between

① $\frac{4}{5}$ and $\frac{3}{4}$ ② $\frac{4}{5}$ and $\frac{5}{6}$ ③ $\frac{4}{5}$ and 0.7 ④ $\frac{3}{4}$, 2

④ Represent the rational numbers: 2, $\frac{-5}{2}$, $\frac{7}{2}$, 0 , -1 on the number line then arrange the ascendingly

⑤ Complete each of the following using the suitable sign (<, > or =):

① $\dfrac{3}{4}$ ... $\dfrac{1}{5}$ 　　② $\dfrac{-3}{15}$ ... $\dfrac{-2}{5}$ 　　③ $\dfrac{3}{15}$ ... $\dfrac{4}{20}$

④ $\dfrac{-6}{9}$ ... $\dfrac{-2}{6}$ 　　⑤ $\dfrac{3}{2}$ ... $\dfrac{3}{5}$

⑥ Complete each of the following

① $\dfrac{3}{5} > \text{...} > \dfrac{2}{5}$ 　　② $\dfrac{-1}{3} > \text{...} > \dfrac{-2}{3}$

③ $\dfrac{1}{4} > \text{...} > \dfrac{1}{8}$ 　　④ $\dfrac{-3}{14} > \text{...} > \dfrac{-2}{7}$

⑤ $\dfrac{1}{4} > \text{...} > \dfrac{1}{6}$ 　　⑥ $\dfrac{-7}{9} > \text{...} > \dfrac{-5}{6}$

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Adding and Subtracting Rational Numbers

1) Adding two rational numbers having the same denominator:

If $\frac{a}{b}$ and $\frac{c}{b}$ are two rational numbers, then:

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

2) Adding two rational numbers with different denominators:

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers then:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

Example ① Add:

1) $\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \frac{3}{5}$

2) $\frac{5}{6} + \left(-\frac{1}{6}\right) = \frac{5-1}{6} = \frac{4}{6} = \frac{2}{3}$ $(\div 2)$

3) $\frac{7}{8} + \left(\frac{-5}{8}\right) = \frac{7-5}{8} = \frac{2}{8} = \frac{1}{4}$ $(\div 2)$

4) $-\frac{1}{4} + \frac{3}{4} = \frac{-1+3}{4} = \frac{2}{4} = \frac{1}{2}$ $(\div 2)$

5) $\frac{5}{9} + \left(-\frac{2}{9}\right) = \frac{5-2}{9} = \frac{3}{9} = \frac{1}{3}$ $(\div 3)$

6) $\frac{-2}{3} + \left(-\frac{1}{3}\right) = \frac{-2-1}{3} = \frac{-3}{3} = -1$

$1\boxed{15}$

(7) $\dfrac{2}{8} \overset{\oplus}{-} \left(-\dfrac{3}{8}\right) = \dfrac{7+3}{8} = \dfrac{10}{8} = \dfrac{5}{4}$

(8) $\dfrac{3}{5} \overset{\ominus}{+} \left(-\dfrac{1}{5}\right) = \dfrac{3-1}{5} = \dfrac{2}{5}$

(9) $\dfrac{1}{34} + \dfrac{3}{4} = \dfrac{1+3}{4} = \dfrac{4}{4} = 1$

(10) $\dfrac{1}{2} + \dfrac{1}{3} = \dfrac{1\times3+1\times2}{2\times3} = \dfrac{3+2}{6} = \dfrac{5}{6}$

(11) $\dfrac{2}{3} + \dfrac{1}{4} = \dfrac{2\times4+3\times1}{3\times4} = \dfrac{8+3}{12} = \dfrac{11}{12}$

(12) $\dfrac{1}{2} + \dfrac{2}{5} = \dfrac{1\times5+2\times2}{2\times5} = \dfrac{5+4}{10} = \dfrac{9}{10}$

(13) $\dfrac{3}{4} - \dfrac{1}{3} = \dfrac{3\times3-1\times4}{4\times3} = \dfrac{9-4}{12} = \dfrac{5}{12}$

(14) $\dfrac{-2}{3} - \dfrac{1}{2} = \dfrac{-2\times2-1\times3}{3\times2} = \dfrac{-4-3}{6} = \dfrac{-7}{6}$

(15) $\dfrac{-1}{5} + \dfrac{1}{3} = \dfrac{-1\times3+5\times1}{5\times3} = \dfrac{-3+5}{15} = \dfrac{2}{15}$

(16) $\dfrac{3}{4} - \dfrac{1}{2} = \dfrac{3\times2-1\times4}{4\times2} = \dfrac{6-4}{8} = \dfrac{2}{8} = \dfrac{1}{4}$

(17) $\dfrac{4}{9} - \dfrac{2}{3} = \dfrac{4\times3-2\times9}{9\times3} = \dfrac{12-18}{27} = \dfrac{-6}{27} = \dfrac{-2}{9}$

properties of addition operation in Q :

① Closure property:

The sum of any two rational numbers is a rational number

If $\frac{a}{b}$, $\frac{c}{d} \in Q$ then

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd} \in Q$$

② Commutative property:

If: $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then:

$$\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$$

③ Associative property:

If: $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers, then:

$$\left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f} = \frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} + \frac{c}{d} + \frac{e}{f}$$

④ The existence of identity element property in addition:

Zero is the identity element in addition operation in Q

$$\frac{a}{b} + Zero = Zero + \frac{a}{b} = \frac{a}{b}$$

(5) the existence of additive inverse property

For every rational number $\frac{a}{b}$ there exist an additive inverse to that is $-\frac{a}{b}$

where $\quad \frac{a}{b} + (-\frac{a}{b}) = zero$

$$-\frac{a}{b} + \frac{a}{b} = zero$$

* Zero is its own additive inverse.

Ex:- Complete

① The additive inverse of $\frac{2}{3}$ is ---

② The additive inverse of $-\frac{1}{2}$ is ---

③ the additive inverse of zero is---

④ the additive inverse of -1 is---

⑤ the additive inverse of $(-\frac{1}{2})^{zero} = ---$

⑥ the additive inverse of $(\frac{3}{5})^{zero} = ---$

⑦ the additive inverse of $|-\frac{3}{5}|$ is ---

Ex: Use the addition properties in Q to carry out the following:

① $\dfrac{2}{7} + \dfrac{3}{4} + \dfrac{5}{7} + \dfrac{1}{4}$

$= \left(\dfrac{2}{7} + \dfrac{5}{7}\right) + \left(\dfrac{3}{4} + \dfrac{1}{4}\right)$

Commutative and associative properties

$= \dfrac{7}{7} + \dfrac{4}{4}$

$= 1 + 1 = 2$ (the result in its simplest form)

② $\dfrac{2}{3} + \dfrac{5}{9} + \dfrac{-2}{3} + \dfrac{4}{9}$

$= \left[\dfrac{2}{3} + \left(-\dfrac{2}{3}\right)\right] + \dfrac{5}{9} + \dfrac{4}{9}$

Commutative and associative properties

$= 0 + \dfrac{9}{9}$ (the additive inverse)

$= \dfrac{9}{9}$ (the identity element)

$= 1$ (the result in its simplest form)

Ex: Write the addition property which used in each of the following

① $\frac{7}{2} + \frac{9}{16} = \frac{9}{16} + \frac{7}{2}$ ② $Zero + \frac{3}{4} = \frac{3}{4}$

③ $\left(\frac{2}{3} + \frac{1}{3}\right) + \frac{1}{6} = \frac{2}{3} + \left(\frac{1}{3} + \frac{1}{6}\right)$

④ $\frac{3}{4} + \left(\frac{-3}{4}\right) = Zero$ ⑤ $\frac{5}{8} + Zero = \frac{5}{8}$

Second: Subtraction operation:

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then

$$\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \left(-\frac{c}{d}\right)$$

i.e. The subtraction operation in Q is defined as adding the minued $\left(\frac{a}{b}\right)$ to the additive inverse of the subtrahend $\left(\frac{c}{d}\right)$

Ex: If $a = \frac{3}{4}$, $b = \frac{-5}{2}$ and $c = \frac{1}{2}$ find the numerical value of each of:

① $a - b$ ② $(a+b) - c$

Solution

① $a - b = \frac{3}{4} - \left(-\frac{5}{2}\right) = \frac{3}{4} + \frac{5}{2}$

$$= \frac{6 + 20}{8} = \frac{26}{8} = \frac{13}{4}$$

② $(a+b) - c$

$$= \left(\frac{3}{4} + \frac{-5}{2}\right) - \frac{1}{2}$$

$$= \left(\frac{6-20}{8}\right) - \frac{1}{2}$$

$$= \frac{-14}{8} - \frac{1}{2} = -\frac{7}{4} - \frac{1}{2} = \frac{-14 - 4}{8} = \frac{-18}{8} = \frac{9}{4}$$

Multiplying and Dividing Rational Numbers

First: Multiplication operation

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers

then: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

Remember:

$$\oplus \times \oplus = \oplus \quad , \quad \ominus \times \ominus = \oplus$$

$$\oplus \times \ominus = \ominus \quad , \quad \ominus \times \oplus = \ominus$$

Ex: Find the result of each of the following in its simplest form:

① $\frac{3}{6} \times \frac{2}{5} = \frac{3 \times 2}{6 \times 5} = \frac{6}{30} = \frac{1}{5}$

② $\frac{-3}{4} \times \frac{2}{9} = \frac{-3 \times 2}{4 \times 9} = \frac{-6}{36} = \frac{-1}{6}$

③ $\frac{1}{2} \times (-2) = \frac{1 \times -2}{2 \times 1} = \frac{-2}{2} = -1$

④ $\frac{-7}{8} \times \frac{4}{14} = \frac{-7 \times 4}{8 \times 14} = \frac{-1 \times 1}{2 \times 2} = \frac{-1}{4}$

⑤ $25\% \times (-\frac{3}{4}) = \frac{25}{100} \times (-\frac{3}{4}) = \frac{1}{4} \times \frac{-3}{4} = \frac{-3}{16}$

⑥ $-1\frac{1}{2} \times -0.\dot{3} = -\frac{3}{2} \times -\frac{1}{3} = \frac{-3 \times -1}{2 \times 3} = \frac{3}{6} = \frac{1}{2}$

properties of the set of rational numbers under multiplication:

for any three rational numbers as $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$

the following properties are satisfied

1) closure property:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} \in Q$$

the product of any two rational numbers is a rational number.

2) Commutative property:

$$\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$$

3) Associative property:

$$\left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} \times \frac{e}{f}$$

4) The existence of multiplicative indentity element property:

The number 1 "one" is the multiplicative identity in Q

$$\frac{a}{b} \times 1 = 1 \times \frac{a}{b} = \frac{a}{b}$$

5) The existence of multiplicative inverse property in Q for every rational number $\frac{a}{b}$ except zero

there is a multiplicative invers $\Rightarrow \frac{b}{a}$

$$\frac{b}{a} \times \frac{a}{b} = 1$$

the multiplicative inverse
property

Ex: Complete

① The multiplicative inverse of the number $\frac{3}{4}$ is ...

② The multiplicative inverse of the number -5 is ...

③ the multiplicative inverse of the number 0.5 is ...

④ the multiplicative inverse of the number $2\frac{1}{2}$ ...

⑤ the multiplicative inverse of the number $(-\frac{2}{3})^{zero}$ ...

⑥ the multiplicative inverse of the number -1 is ...

Remarks

① The multiplicative inverse of the rational number is called the reciprocal of the rational number

② Zero has no multiplicative inverse

$$\frac{1}{0} \text{ is meaningless}$$

③ multiplying any rational number by zero equals zero

6 The distributive property :

Ex: Use the distributing property to find the value of each of the following:

1) $\dfrac{13}{12} \times 7 + \dfrac{13}{12} \times 5$

$$= \dfrac{13}{12} \times (7+5) = \dfrac{13}{12} \times \dfrac{12}{1} = 13$$

② $\dfrac{4}{9} \times 11 + \dfrac{4}{9} \times 16$

$$= \dfrac{4}{9} \times (11+16) = \dfrac{4}{9} \times \dfrac{27}{1}^{3} = 12$$

3) $\dfrac{5}{13} \times 2 + \dfrac{5}{13} \times 4 + \dfrac{5}{3} \times 7$

$$= \dfrac{5}{13} \times (2+4+7) = \dfrac{5}{13} \times 13 = 5$$

4) $\dfrac{4}{7} \times 4 + \dfrac{4}{7} \times 2 + \dfrac{4}{7} \times 1$

$$= \dfrac{4}{7} \times (4+2+1)$$

$$= \dfrac{4}{7} \times 7 = 4$$

⑤ $\dfrac{-3}{7} \times 8 + 5 \times \left(\dfrac{-3}{7}\right) + \left(\dfrac{-3}{7}\right)$

$$= \dfrac{-3}{7} \times (8+5+1) = \dfrac{-3}{7} \times \overset{2}{14} = -6$$

Second: Division operation.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

Ex: Find the result of each of the following in the simplest form:

① $\frac{1}{2} \div \frac{3}{5} = \frac{1}{2} \times \frac{5}{3} = \frac{1 \times 5}{2 \times 3} = \frac{5}{6}$

② $\frac{-3}{5} \div \frac{9}{5} = \frac{-3}{5} \times \frac{5}{9} = \frac{-15}{45} = \frac{-1}{3}$

③ $\frac{-14}{15} \div \left(\frac{-21}{5}\right) = \frac{-14}{15} \times \frac{-5}{21} = \frac{-2}{3} \times \frac{-1}{3} = \frac{2}{9}$

④ $2\frac{1}{4} \div 1\frac{1}{2} = \frac{9}{4} \div \frac{3}{2} = \frac{9}{4} \times \frac{2}{3} = \frac{18}{12} = \frac{3}{2}$

Ex: find the value of x in each of the following

① $\frac{5}{7} \times x = \frac{5}{7}$

② $x \times \frac{17}{3} = 1$

③ $\frac{-7}{3} \times x = zero$

④ $\frac{-7}{3} \times \frac{-3}{7} = x$

⑤ $\frac{3}{5} \times x = \frac{-4}{5} \times \frac{3}{5}$

⑥ $\frac{3}{4} \div x = \frac{3}{4} \times \frac{-2}{5}$

Applications on the Rational Numbers

[1] The number that lies at the midpoint of the way between any two numbers

$$= \frac{1}{2} \text{ Sum of them}$$

[2] the number that lies at one third of the way between two numbers

$$= \text{The smallest number} + \frac{1}{3} \text{ the distance}$$

$$= \text{the greatest number} - \frac{1}{3} \text{ the distance between the two numbers}$$

Ex.1 Find the number that lies at the midpoint of the way between $\frac{1}{2}$, $\frac{1}{3}$

Solution

$$\frac{1}{2} \,,\, \frac{1}{3} \Rightarrow \frac{3}{6} \,,\, \frac{2}{6}$$

the Sum $= \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

The number that lies at the midpoint of the way between $\frac{1}{2}$, $\frac{1}{3} = \frac{1}{2}$ sum

$$= \frac{1}{2} \times \frac{5}{6}$$

$$= \frac{5}{12}$$

Ex: Find a rational number lying at one fourth of the way between $\frac{1}{2}$, $\frac{1}{3}$

Solution

$$\frac{1}{2}, \frac{1}{3} \Rightarrow \frac{3}{6}, \frac{2}{6}$$

the distance $= \left| \frac{3}{6} - \frac{2}{6} \right| = \frac{1}{6}$

the required number =

the smaller number $+ \frac{1}{4}$ distance

$$= \frac{2}{6} + \frac{1}{4} \times \frac{1}{6}$$

$$= \frac{2}{6} + \frac{1}{24} = \frac{8+1}{24} = \frac{9}{24} = \frac{3}{8}$$

Ex: Find a rational number lying at one fifth of the way between $\frac{2}{5}$ and $\frac{4}{7}$

Solution $\quad \frac{2}{5}, \frac{4}{7} \Rightarrow \frac{14}{35}, \frac{20}{35}$

the distance $= \left| \frac{20}{35} - \frac{14}{35} \right| = \frac{6}{35}$

the required number =

the smaller number $+ \frac{1}{5}$ distance

$$= \frac{14}{35} + \frac{1}{5} \times \frac{6}{35} = \frac{14}{35} + \frac{6}{175} = \frac{70+6}{175} = \frac{76}{175}$$